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ON TIDE CURRENTS IN ESTUARIES AND RIVERS.*

BY ERNEST W. BROWN.

IN popular and elementary text-books where the phenomena exhibited by the tides are described, a statement is frequently made which almost invariably causes difficulty because it appears to contradict every-day experience. This difficulty arises partly from insufficient explanation and partly from a want of completeness in giving the conditions which must hold in order that it may be true. In many cases of wave motion it is easier to give explanation by means of a diagram showing the form of the wave rather than the curves described by the individual particles which partake of the wave motion. In certain cases of tide waves, however, I shall try to show how the explanations can be considerably simplified by considering the motion of a single drop of the water or, let us say, of a float on the surface.

The particular statement referred to above is that in rivers, slack water (that is, no current) at any place occurs when the water is at mean level, while the maximum flow (upward current) occurs at high water and maximum ebb (downward current) at low water. This, of course, is a purely theoretical result and is quite contrary to what is usually noticed, namely, that slack water occurs very near high and low water. It demands that the bed of the river shall not be uncovered at any time, that its breadth and sectional area shall be uniform, that friction be neglected, that there be no proper current of the river and that the wave shall not change its shape as it proceeds, the last condition not holding even theoretically. Moreover, the observations must be made at places on the canal very far distant from its mouth. The place of observation is the principal cause of the difference between the theoretical and the observed results. The tidal currents are usually noticed comparatively near the mouth; if we go further up the river so as to be free from this difficulty, the friction, change of shape of the bed, etc., play important parts, and a great difference between the theoretical and observed motions still remains.

When a wave passes over the surface of still water, each particle of water describes a closed curve whose shape depends on that of the wave. With ordinary waves on deep water raised, for example, by a ship, this curve is an oval nearly symmetrical about both vertical and horizontal axes, as a little observation on a float will easily show. With ordinary sea waves in deep water the vertical axis is much greater than the horizontal axis. Near the beach, that is, in shallow water, the reverse takes place, and what is noticed mainly is the motion of the float to and fro rather than up and down.

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* This paper was read before the American Mathematical Society at the meeting of April 29, 1899.

Consider a straight uniform canal with vertical sides and horizontal bed in which the water is sufficiently deep so that the waves we shall consider do not leave it dry at any time. A series of similar waves passing along the length of the canal will cause a float to describe a nearly symmetrical oval curve. If the wave length (distance from crest to crest) is not long compared with the depth of the water, the float will have but little horizontal motion. As the wave length increases, the height remaining the same, the horizontal motion will increase. When the height of the wave is a few feet, the depth of the canal a few fathoms and the wave-length several miles, a person standing on the bank notices mainly the horizontal motion. If, however, he follow the float by walking along a horizontal bank, he will notice that the float is gradually rising or falling. The rule is that the water is flowing in the direction of the wave motion when the float is above its mean level, and in the opposite direction when it is below its mean level. How this takes place will be seen at once from fig. 1.

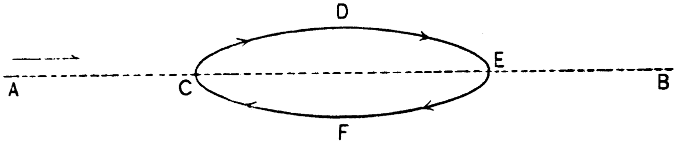

Fig. 1.

Here *A B* represents the mean level of the water in the direction of the length of the canal, *C D E F* the curve described by the float. When it is at *D* there is high water at that part of the canal, with maximum current in the direction of the arrow on the left which shows the direction of motion of the wave. At *C* and *E* we have slack water and mean level; and so on.

Suppose now the canal communicates with the sea at one end. The tidal motion in the canal is mainly caused by the tides of the ocean, that is, by a rise and fall of the ocean at the mouth of the canal. At a great distance from the mouth, the motion in our canal would still be like that which has just been described, the length *C E* being some miles and *D F* a few feet. If the ocean be deep and the tide wave advance in a direction perpendicular to the cliffs, the motion of a float some distance from the land will be mainly vertical. Nearer the mouth the horizontal motion begins to be perceptible; and just inside it becomes considerable. If we have a series of floats extending from far out into the ocean right up the canal, the curves described by them vary from ovals with horizontal axes of a few feet to ovals with horizontal axes of several miles, the vertical axes being never more than a few feet. Moreover, the ovals described by floats near the mouth are no longer symmetrical, and the axes depart from the horizontal and vertical.

In fig. 2, AMB represents mean level, AM in the ocean and MB in the canal. The points D, F represent high and low water respectively in the several positions; the points E and C slack water. Thus near the mouth slack water occurs very near high and low water. This agrees with observation. Moreover, it is observed that the current "flows" some time after high water and "ebbs" some time after low water; this is illustrated in the parts DE and FC of the curves II, III, and IV the water falling and flowing from D to E and rising and ebbing from F to C; from C to D we have rising water with flowing current, and from E to F falling water with ebbing current.

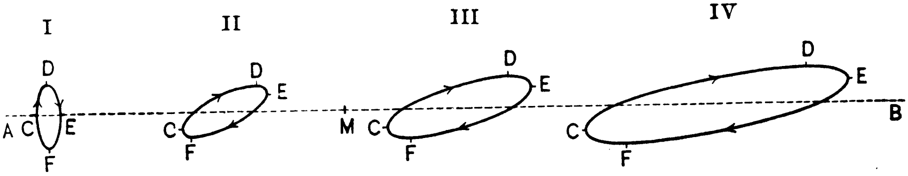

Fig. 2.

This general effect is modified by other circumstances to a very considerable extent in some rivers. Most rivers gradually decrease in depth and breadth as we get farther away from the mouth, causing (as theory indicates) an increased height of the wave and alteration of its length as it progresses up the river. In general the motion of the float at the end of the longer axis in curves III and IV is very slow compared to that at the ends of the shorter axis. In some rivers however this is not so. At C it may become very quick when it gives rise to the "bore," which takes the appearance of a bank of water, sometimes several feet in height, travelling up the river and causing the water at the places it passes to rise suddenly. The curve described has also lost all symmetry. All these phenomena are fully described in Professor Darwin's book, *The Tides*. The reader will be able to find the shape of the curves described by floats in such cases for himself.

There is no attempt at accuracy in the diagrams. These are merely intended to give a general idea of the relation of height to ebb and flow. The vertical distances are of course much exaggerated.

HAVERFORD COLLEGE, NOVEMBER, 1899.

NOTE ON NETTO'S THEORY OF SUBSTITUTIONS.

BY G. A. MILLER.

IN finding a function that belongs to a given group, Netto employs a method which is apt to mislead the student. Since some other authors express themselves somewhat indefinitely* in regard to this fundamental problem, it may be desirable to give several elementary illustrations which will exhibit the points in question. We shall first show how the given method may lead to incorrect results. Employing the same Galois function ($\phi_1 = x_1 + ix_2 - x_3 - ix_4$) as Netto employs† we proceed to find a function belonging to the group $G_2 = 1, (x_1x_2)(x_3x_4)$ by the first method which he employs to find a function belonging to the group

$$G_8 = 1,\ (x_1x_2)(x_3x_4),\ (x_1x_3)(x_2x_4),\ (x_1x_4)(x_2x_3),\ (x_1x_3),\ (x_2x_4),\ (x_1x_2x_3x_4),\ (x_1x_4x_3x_2).$$

Transforming ϕ_1 by the substitutions of G_2 we have:

$$\psi_1 = \phi_1\phi_2 = (x_1 + ix_2 - x_3 - ix_4)(x_2 + ix_1 - x_4 - ix_3) = i(x_1 - x_3)^2 + i(x_2 - x_4)^2.$$

If we transform ϕ_1 by the substitutions of $G'_2 = 1, (x_2x_4)$ we obtain:

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* Vogt, *Résolution algébrique des équations*, 1895, p. 23.

† Netto, *Theory of Substitutions*, translated by Cole, 1892, p. 30.